CONVECTIVE HEAT TRANSFER IN CONDITIONS OF A TRANSIENT BOUNDARY LAYER AT A CYLINDER IN A TRANSVERSE FLOW

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A finite-difference method is proposed for solving the equations of the thermal boundary layer at a circular cylinder, and the results of calculating the heat transfer in transient flow conditions are presented.

As is known [1], laminar, transient, and turbulent zones are formed when $Re > 2 \cdot 10^5$ at a cylinder in a transverse flow, and breakaway of the flow occurs. Theoretical investigation of the heat transfer is complicated by the presence of the transient and turbulent transfer, the mechanism of which has not been adequately studied and has only been subjected to approximate modeling on the basis of experimental data. The flow and heat transfer at the cylinder at large Re and various Pr have not been adequately studied, and require further investigation.

In the present work, based on an analysis of the experimental data of [1], a method is proposed for numerical solution of the equations of the thermal boundary layer with regions of various flow conditions, taking account of the influence of variable physical properties, the degree of turbulence of the flow, and clogging of the channel for heat transfer of the cylinder in a transverse flow of viscous liquid; the corresponding results are presented.

Basic Equations

The following transformation of the transverse variable [2] is used to increase the accuracy of numerical non-self-similar solution of the equations of an incompressible thermal boundary layer [1]

$$y \to \omega \to z \to \xi, \tag{1}$$

where $\omega = \psi/\psi_f$, ψ is the flow function

$$\psi = \Big(\int_0^y \rho u dy\Big)_{x=\mathrm{const}}, \tag{2}$$

satisfying the continuity equation

$$z = \sqrt{\omega},\ 0 \leqslant \omega,\ z \leqslant 1, \tag{3}$$

$$\xi = A \ln(1 + z/\beta),\ 0 \leqslant \xi \leqslant 1. \tag{4}$$

It follows from Eq. (4), when $\xi = 1$, that $A = 1/\ln(1 + 1/\beta)$.

Using the variables x, ξ, the initial boundary-layer equations [1] are written in the following form

$$\frac{\partial u}{\partial x} + B_1 b\frac{\partial u}{\partial \xi} = B\frac{\partial}{\partial \xi}\left(Bc_u\frac{\partial u}{\partial \xi}\right) + \frac{\rho_f U}{\rho u}\frac{dU}{dx}, \tag{5}$$

$$\frac{\partial T}{\partial x} + B_1 b\frac{\partial T}{\partial \xi} = \frac{B}{c_p}\frac{\partial}{\partial \xi}\left(Bc_T\frac{\partial T}{\partial \xi}\right), \tag{6}$$

where

$$b = (\rho v/\psi)_f,\ B = \frac{\partial \xi}{\partial \omega} = \frac{A}{2z(\beta + z)'},\ B_1 = Bz^2, \tag{7}$$

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$$c_u = \rho u(\mu + \gamma\mu_t + \mu_{Tu})/\psi_f^2, \; c_T = \rho u(\lambda + \gamma\lambda_t + \lambda_{Tu})/\psi_f^2. \quad (8)$$

The boundary conditions for Eqs. (5) and (6) are as follows:

$$u(x, 0) = v(x, 0) = 0, \; T(x, 0) = T_w \text{ or } q(x, 0) = q_w, \quad (9)$$

$$u(x, 1) = U, \; T(x, 1) = T_f. \quad (10)$$

The modified Khiments formula is used for closure of Eqs. (5) and (6) with the boundary conditions in Eqs. (9) and (10), taking account of clogging of the channel [1] in the case of a turbulized laminar boundary layer and the experimental distribution with a turbulent boundary layer, especially for the region $\varphi > 80\text{-}140°$, i.e., before breakaway of the turbulent flow [1, 3].

The turbulent-transfer coefficients in the boundary layer μ_t and λ_t are determined from the path length of Prandtl mixing [2], while the influence of turbulence of the incoming flow on the boundary-layer characteristics is taken into account by the empirical dependences of μ_{Tu} and λ_{Tu} given in [1].

The factor corresponding to transition from laminar to turbulent flow conditions in Eq. (8) ($0 < \gamma \le 1$) is expressed as follows [4]

$$\gamma = 1 - \exp\left[-\frac{G}{U}(x - x_{tr})^2\right], \quad (11)$$

where $G = (3/C^2)(U^3/\nu^2)\mathrm{Re}_{x_{tr}}^{-1.34}$, $C = 60 + 4.86\,M^{1.92}$ when $0 < M < 5$.

The onset of transient flow conditions at the cylinder and x_{tr} in Eq. (11) are determined under the condition

$$(\mu_t + \mu_{Tu})_{max}/\mu = 10, \quad (12)$$

obtained on the basis of numerical experiment and analysis of the experimental data of [1, 3]. When $x > x_{tr}$, $0 < \gamma \le 1$.

The longitudinal curvature of the body in the flow (in the present case, the cylinder diameter d = 30-70 mm) is taken into account only in the transient and turbulent flow conditions [1].

The point of flow breakaway from the cylinder is determined from the condition

$$(\partial u/\partial y)_w = 0. \quad (13)$$

In calculating the turbulent flow ($\mathrm{Re} > 2\cdot10^5$), the breakaway condition in Eq. (13) is not always satisfied. In this case, the point of breakaway is determined on the basis of the condition of growth in local surface friction with positive pressure gradient. There are also other indirect methods of determining the point of breakaway of the turbulent flow [5, 6].

According to the Stratford method [5], the point of flow breakaway in the present case is determined from the relation

$$F(x) = P^+\left[(x_m - x)\frac{dP^+}{dx}\right]^{1/2}\left(\frac{10^{-6}}{\mathrm{Re}_m}\right)^{-1/10}, \quad (14)$$

where

$$P^+ = 1 - \left(\frac{U}{U_m}\right)^2, \; \mathrm{Re}_m = \frac{(x_m - x)U_m\rho_f}{\mu_f}$$

when $P^+ \le 4/7$; $x_m > x_{tr}$.

In solving Eq. (14), x_m, U_m, x_{tr}, and U_{tr} must first be determined, theoretically or experimentally. The numerical values of F(x) are analyzed for the proposed breakaway zone of the turbulent flow ($110 < \varphi < 140°$): if $F(x) > 0.40$, breakaway occurs when $F(x) = 0.40$, and if $F(x) < 0.40$ at the maximum of F(x).

As well as Eq. (14), it is expedient to use the boundary-layer form parameter $H = \delta_1/\delta_2$, which is simple to do in solving the boundary-layer equation. It is assumed that, with

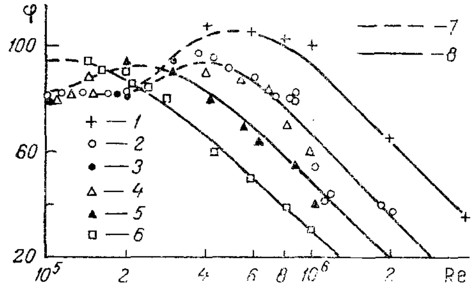

Fig. 1

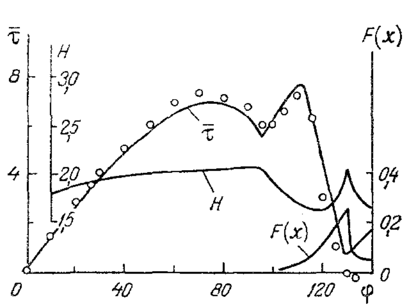

Fig. 2

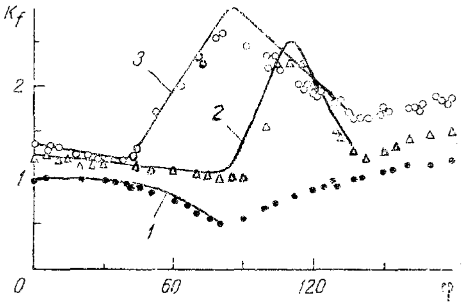

Fig. 3

Fig. 1. Dynamics of the points of onset of transition from a laminar to a turbulent boundary layer: 1) experiment [3]; 2-6) [1]; 7) calculation [7]; 8) calculation by the present method; 1) Tu = 0.45; 2-4) 1.0; 5) 7.0; 6) 15.0%.

Fig. 2. Variation in tangential stress $\bar{\tau} = 2\tau_w\sqrt{Re}/(\rho U_\infty)$ above the cylinder when $Re = 1.27\cdot10^6$ and form parameters F(x) in Eq. (14) and $H = \delta_1/\delta_2$ in the breakaway zone of the turbulent flow; the curves correspond to calculation and the points to experiment [3].

Fig. 3. Results of calculating local heat transfer $K_f = Nu_f Re^{-0.5} Pr_f^{-0.33}(Pr_f/Pr_w)^{-0.25}$ of a cylinder in a water flow, and comparison with experimental data [1]: 1) $Re = 5.5\cdot10^4$; 2) $8.6\cdot10^5$; 3) $2.03\cdot10^6$; Tu ≈ 1.5%.

with rapid growth in H, breakaway of the turbulent flow may be expected on a short section up to values of 1.8-2.6 [6].

The finite-difference method is used for numerical solution of Eqs. (5) and (6). The difference approximation of these equations is obtained by integrating around a small control area surrounding the difference point [2].

The method employed was described in more detail in [1, 2].

Calculation Results

The flow and heat transfer at a cylinder when $10^3 \leq Re \leq 10^7$ and $0.7 \leq Pr \leq 300$ may be determined by the above method from the frontal critical point ($\varphi \simeq 1°$) to the point of breakaway of the laminar [7] or turbulent boundary layer determined from Eqs. (13) and (14).

With transient flow conditions, a laminar boundary layer is formed at the frontal section of the cylinder; in modeling this boundary layer ($\gamma = 0$), some dependences for practical application are obtained.

The influence of variable physical properties of the liquid on the heat transfer in the calculational data is generalized by the dependence

$$Nu/Nu_0 = (T_f/T_w)^{0,36(n_\rho + n_\lambda)a} \quad (15)$$

TABLE 1. Transient Number $Re_1 = U\delta_1/\nu$ as a Function of Re and Tu

$Re \cdot 10^5$	Tu, %			
	0,45	1,0	7,0	15,0
2	—	—	660	520
4	—	900	660	530
6	1270	860	700	460
8	1350	850	670	380
10	1500	840	530	370
20	1100	—	—	—

with air or gas flow around the cylinder when $Pr \simeq 0.7$; $a = 1.0$ when $T_f > T_w$, $a = 0.5$ when $T_f < T_w$; n_ρ and n_λ are the exponents in dependences of the type $\psi/\psi_f = (T/T_f)^n$ for determining the variable physical properties of the heat carrier [8]. In flow of viscous liquids around the cylinder ($3 \le Pr \le 300$)

$$Nu/Nu_0 = (Pr_f/Pr_w)^{m_1}, \tag{16}$$

where $m_1 = 0.25$ when $T_w > T_f$ and $m_1 = 0.20$ when $T_w < T_f$.

The influence of Tu and k_q on the local heat transfer may be expressed as follows

$$\frac{Nu_x(Tu \neq 0)}{Nu_x(Tu = 0)} = Tu^{0.17}, \tag{17}$$

$$\frac{Nu_x(k_q \neq 0)}{Nu_x(k_q \to 0)} = 0.86(1 + 1.18k_q^3), \tag{18}$$

which are confirmed by experimental data [1].

Calculation of the flow and heat transfer in the transient and turbulent zones ($0 < \gamma \le 1$) is less accurate and more difficult than in the laminar zone ($\gamma = 0$).

Calculations of x_{tr} (or φ_{tr}) corresponding to Eq. (12) show that increase in Re and Tu (Fig. 1) facilitates earlier transition to turbulent flow. The results of calculating Re_1 for practical use in Table 1 correspond to the data in Fig. 1.

In the case of variable physical properties of viscous liquids ($3 \le Pr \le 300$), the values for Eq. (12) agree with the available literature data on the onset of transition and may be generalized by the following dependence, with a relative error of ±15%

$$\varphi_{tr} = \varphi_{tr,0} + 0.37 \ln(Pr_f/Pr_w), \text{ rad.} \tag{19}$$

As is evident from Fig. 2, where the variation in $\bar{\tau}$, H, and F(x) characteristic for transverse flow around a circular cylinder when $Re > 2 \cdot 10^5$ is shown, the breakaway condition in Eq. (13) does not hold but, judging from all three parameters, according to the above theory, it may be asserted that $\varphi_{br} \simeq 130°$. The experimental data of [3] show that $\varphi_{br} \simeq 128°$.

The most complex process is heat transfer from the transient and turbulent flow zones (Fig. 3), which may be modeled with acceptable accuracy by the given method from $\varphi = 1°$ to breakaway of the turbulent flow, i.e., to $\varphi \simeq 130°$.

The present method does not take account of effects such as the previous history of the flow or the inverse influence of the interrupted flow on the other characteristics; this requires more profound theoretical investigation using the Navier–Stokes equation [9].

NOTATION

C_p, specific heat, J/kg·K; d, cylinder diameter, m; h, channel height, m; k_q, k, degree of channel clogging (d/h); q, specific heat flux, W/m²; T, temperature, K; u, v, components of mean velocity, m/sec; U, maximum velocity at external boundary of boundary layer, m/sec; U_∞, velocity of incoming flow, m/sec; x, y, coordinates along and perpendicular to cylinder perimeter, m; δ, boundary-layer thickness, m; δ_1, displacement thickness, m; δ_2, thickness of momentum loss, m; α, heat-transfer coefficient, W/m·K; λ, thermal conductivity, W/m·K; μ, dynamic viscosity, N·sec/m²; ν, kinematic viscosity, m²/sec; ρ, density, kg/m³;

τ, tangential stress, N/m^2; $\bar{\tau}$, dimensionless tangential stress, $[2\tau_w\sqrt{Re}/(\rho U_\infty)]$; φ, reference angle from frontal critical point of cylinder, deg; M, Mach number; Nu_f, Nusselt number $(\alpha d/\lambda)$; Pr, Pr_f, Prandtl number $(\mu_f c_{pf}/\lambda_f)$; Re, Reynolds number $(U_\infty d\rho_f/\mu_f)$; Tu, turbulence of flow $(100\sqrt{u'^2}/U_\infty)$, %. Indices: f, external boundary of boundary layer, $y \geq \delta$; 0, isothermal conditions; ∞, unperturbed flow, m, point of pressure minimum; tr, point of onset of transient flow conditions; t, turbulent layer; Tu, with turbulence of the incoming flow; T, heat-conduction equation.

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FRICTION AND HEAT EXCHANGE IN FLOW OVER A PERMEABLE SURFACE

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The extremal character of the dependence of friction on suction velocity on a permeable surface immersed in an incompressible liquid flow is established. The suction value corresponding to maximum friction and the limiting heat exchange intensity are calculated.

Interest in the study of transport processes in flow over surfaces made of permeable materials has been stimulated by a number of practical technological applications in both traditional (air and water transport), and new fields of contemporary industry.

Recent studies have shown that the efficiency of drying of sheet and roll materials is increased significantly by thermal processing with a jet draft of heated gas [1]. However, introduction into practice of the progressive techniques realized by this method [2, 3] and development of corresponding methods for calculating equipment parameters [4] demand an ever-increasing understanding of the physical bases of the transport processes involved.

The goal of the present study is to analyze the physical features of thermal and dynamic interaction of an incompressible flow with a permeable surface of a body over which the flow passes. Quantitative data were obtained on hydrodynamics, friction, and heat exchange in the presence of intense surface fluxes of matter. These data can be used independently,

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